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Engineering Notes

Optimal Formation Reconfigurations Subject to Hill Three-Body Dynamics

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I. Introduction

THIS paper presents fuel-optimal continuous thrust formation flying maneuvers near the L_2 Lagrangian point of the Hill restricted three-body dynamics. A recently developed technique based on generating functions [1,2] is applied to determine optimal cost as an algebraic function of prescribed boundary conditions and develop optimal control in feedback form. This analytic nature provides a more computationally tractable procedure than the conventional shooting technique, especially when it is necessary to analyze multiple spacecraft maneuvers for many cases of boundary conditions and time spans.

The comprehensive literature on relative motions and formation flying maneuvers is substantial, and could result in a stand-alone
2 survey paper; thus, only some works in recent years are cited [1,3–18]. Some researchers [13,15,17] employ numerical methods to study optimal reconfiguration problems, and others [3–7,9–12, 14,16,18] seek to derive analytical solutions to various forms of relative dynamics and/or analyze formation flying maneuvers. Whereas [3,4,9,10,12,14,16] focus on linearized dynamics of relative motions, [5–8,13,15,18] deal directly with higher-order dynamics or introduce nonlinearities to extend the applicability of linearized dynamics.

Mission analysis and design related to formation flying transfers usually require the development of state and control trajectories for
3 numerous pairs of boundary conditions and operational time spans. For example, Guibout and Scheeres analyzed about eight million cases of formation flying transfers in order to properly design a virtual mission near the Earth's gravitational field [8]. Hence, solutions in analytic and explicit form are preferred for the sake of computational reliability and efficiency. At the same time, highly accurate solutions reflecting realistic situations can be obtained by introducing higher-order dynamics or nonlinearity. These observations motivated to employ the semi-analytic nature of generating functions to analyze optimal formation flying maneuvers subject to the Hill three-body dynamics.

Under mild assumptions of the analyticity of the performance index and dynamical system, the generating function approach provides the initial adjoint and the optimal control in series form, with respect to known boundary conditions and time index. It is numerical in the sense that the initial value problems are solved for a system of ordinary differential equations numerically, and is analytic in the sense that the solution in explicit series form can be directly evaluated for desired boundary conditions and time spans. Because the order of Taylor expansion can be set as high as desired and is only limited by computer memory capacity, the applicability of the generating function is not affected by the complexity of dynamic environments or of formation configuration.

The discussion begins with formally defining an optimal control problem for the relative motion near the L_2 Lagrangian point of the Hill restricted three-body problem (Sec. II). Accuracy of higher-order solutions is demonstrated first. Optimal cost analysis for multiple spacecraft maneuvers and comparative study between continuous and impulsive thrust are presented (Sec. III). Finally, the conclusion follows (Sec. IV).

II. Optimal Formation Reconfiguration Problem

Consider motions of multiple spacecraft in formation subject to the Hill restricted three-body dynamics. It is assumed in the Hill three-body problem that the first body has a considerably larger mass than the second body, the third body (usually a spacecraft) has a negligible mass, and the coordinate origin is located at the second body [19,20]. Once properly scaled, the equations of motion can be written as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} X_2 \\ 3X_1 + 2X_4 - \frac{X_1}{R^3} + U_1 \\ X_4 \\ -2X_2 - \frac{X_3}{R^3} + U_2 \\ X_6 \\ -X_5 - \frac{X_5}{R^3} + U_3 \end{bmatrix}, \qquad R = \sqrt{X_1^2 + X_3^2 + X_5^2} \tag{1}$$

where (X_1, X_3, X_5), (X_2, X_4, X_6), and (U_1, U_2, U_3) represent the position, velocity, and control components, respectively. It is well known that there exist two equilibrium points $[(\pm 3)^{-1/3} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$ called the Lagrangian points. As a specific problem of interests, formation flying maneuvers near the L_2 Lagrangian point is considered. If the actuation-free reference trajectory is chosen as

$$\boldsymbol{X}_0(t) \equiv [(+3)^{-1/3} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \qquad \boldsymbol{U}_0(t) \equiv [0 \quad 0 \quad 0]^T \tag{2}$$

then the associated equations of relative motions $\{\boldsymbol{x}(t), \boldsymbol{u}(t)\}$ can be obtained by expanding Eq. (1) with respect to Eq. (2)

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{g}(x(t), \boldsymbol{u}(t), t) \tag{3}$$

It is assumed that the spacecraft has a low-thrust, power-limited propulsion system for which the fuel consumption is proportional to the integral of the scalar product of thrust accelerations over time [4].

Problem 1 (Fuel-optimal reconfiguration maneuvers near the L_2 Lagrangian point): Minimize the quadratic performance index

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